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Nonlinear dynamics of feedback modulated magnetic islands[a]

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Introduction

- Development of reliable method for controlling tearing mode amplitudes is key to further progress in both advanced tokamaks and RFPs.
- Active control via externally applied, rotating, helical magnetic perturbations is a promising candidate which is currently being evaluated on HBT-EP.

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General approach

- Aim of our study is to develop pair of ordinary differential equations governing time evolution of both helical phase and radial width of magnetic island chain in presence of externally generated, resonant magnetic perturbation.
- Our investigation takes place within context of standard zero-β, cylindrical, resistive-MHD theory.

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Ion polarization current

- Even within restrictive context of zero-β, cylindrical, resistive-MHD theory, there is still considerable controversy regarding form of equations governing island dynamics.
- Main point of contention is effect of ion polarization current—due to plasma flow around island chain—on island growth.
- It has been reported by many authors[a][b][c][d] that ion polarization effect is stabilizing.

[a] R. Fitzpatrick, and T.C. Hender, Phys. Fluids B **3**, 644 (1991).
[b] A.I. Smolyakov, Plasma Phys. Control. Fusion **35**, 657 (1993).
[c] M. Zabiego, and X. Garbet, Phys. Plasmas **1**, 1891 (1994).
[d] H.R. Wilson, *et al.*, Phys. Plasmas **3**, 248 (1996).

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Regimes of operation—I

- We have identified three distinct regimes of operation, depending on magnitude of modulation frequency: *i.e.*, instantaneous difference between helical phase velocity of island chain and external perturbation.
- In non-localized regime,

$$|v'|\,\tau_V \ll 1,$$

perturbed velocity profile extends across whole plasma. Here, τ_V is global viscous diffusion time-scale.

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Regimes of operation—II

- In weakly localized regime,

$$1 \ll |v'|\,\tau_V \ll (r_s/W)^2,$$

perturbed velocity profile localized in vicinity of island chain, but width of profile still greatly exceeds island width.

- In strongly localized regime,

$$(r_s/W)^2 \ll |v'|\,\tau_V,$$

perturbed velocity profile collapses to thin boundary layer on island chain separatrix, plus residual profile which extends few island widths beyond separatrix. Here, r_s is radius of rational surface, and W is island width.

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Non-localized regime—I

- Rutherford equation:

$$\tau_R\,\frac{d(W/r_s)}{dt} = E + E_{sc}\left(\frac{W}{W_c}\right)^2\cos\varphi + \frac{l_1}{8\sqrt{2}}\,E_{sc}^2\,K_\phi\,\frac{\tau_V^2}{\tau_H^2}\left(\frac{W}{4\,r_s}\right)^3\left(\frac{W_c}{4\,r_s}\right)^4\sin^2\varphi.$$

- τ_R is resistive diffusion time-scale: E is linear stability index: E_{sc} is coupling coefficient between mode and external perturbation: W_c is measure of amplitude of external perturbation: τ_H is hydromagnetic time-scale: K_ϕ is correction for poloidal flow damping: φ is helical phase of mode w.r.t. external perturbation.

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Non-localized regime—II

- First term on r.h.s. is linear drive: second term is modification to stability due to external perturbation: third term is ion polarization effect.
- Velocity integral $l_1 = 0.3319$ is positive. Hence, ion polarization effect is destabilizing in non-localized regime. Consistent with our previously published result[a].

[a] F.L. Waelbroeck, and R. Fitzpatrick, Phys. Rev. Lett. **78**, 1703 (1997).

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Non-localized regime—III

- Equation of motion:

$$J_1\,\tau_V^2\,\frac{dv}{dt}+\frac{\tau_V\,(v-v_0)}{J}+\frac{E_{sc}\,K_\phi}{2}\,\frac{\tau_V^2}{\tau_H^2}\left(\frac{W}{4\,r_s}\right)^2\left(\frac{W_c}{4\,r_s}\right)^2\sin\varphi=0.$$

- J and J_1 are dimensionless integrals involving perturbed velocity profile across whole plasma: v is helical phase velocity of island chain: v_0 is unperturbed helical phase velocity of island chain.
- Equation of motion takes form of pendulum equation. First term is inertia of plasma co-rotating with island: second term is viscous restoring torque: third term is electromagnetic torque due to external perturbation.

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Weakly-localized regime

- Rutherford island equation takes same form as in non-localized regime.
- Equation of motion:

$$\begin{aligned}\sqrt{\frac{2}{|v'|\,\tau_V}}\,\tau_V^2\,\frac{dv}{dt}+\sqrt{2\,|v'|\,\tau_V}\,\tau_V\,(v-v_0)&\\+\frac{E_{sc}\,K_\phi}{2}\,\frac{\tau_V^2}{\tau_H^2}\left(\frac{W}{4\,r_s}\right)^2\left(\frac{W_c}{4\,r_s}\right)^2\sin\varphi&=0.\end{aligned}$$

- Inertia term smaller than before, since co-rotating region of plasma has shrunk. Viscous restoring term larger than before, since radial velocity scale-length has shrunk.

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Strongly-localized regime—I

- Rutherford equation:

$$\tau_R \frac{d(W/r_s)}{dt} = E + E_{sc}\left(\frac{W}{W_c}\right)^2 \cos\varphi + 2\sqrt{2}\, l_2 \left(\frac{4\, r_s}{W}\right)^3 \frac{(v-v_0)^2\, \tau_H^2}{K_\phi}.$$

- Velocity integral $l_2 = 0.4875$ is positive, indicating that ion polarization effect is destabilizing in strongly-localized regime.

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Strongly-localized regime—II

- Equation of motion:

$$l_3 \left(\frac{W}{4\, r_s}\right) \tau_V^2 \frac{dv}{dt} + l_4 \sqrt{2\,|v'|\,\tau_V}\, \tau_V\, (v-v_0) + \frac{E_{sc}\, K_\phi}{2} \frac{\tau_V^2}{\tau_H^2} \left(\frac{W}{4\, r_s}\right)^2 \left(\frac{W_c}{4\, r_s}\right)^2 \sin\varphi = 0.$$

- Velocity integrals $l_3 = 2.758$ and $l_4 = 5.130$ are positive, ensuring that inertial and viscous restoring terms have sensible signs.

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Strongly-localized regime—III

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- All previously published calculations effectively performed in strongly-localized regime.
- However, these calculations erroneously neglect effect of viscous boundary layer on island separatrix.
- Neglect of boundary layer has three consequences:
  - Ion polarization term becomes stabilizing.
  - Viscous restoring term becomes proportional to $1/W$.
  - Inertial term becomes negative—would give rise to insane island dynamics!

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Summary

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- Have obtained analytic expressions for perturbed velocity profile, Rutherford equation, and island equation of motion in three different asymptotic regimes.
- Ion polarization term is destabilizing in each regime. Previous reports that this effect is stabilizing are in error.
- Preliminary attempts to use mode equations to simulate actual feedback experiments have yielded promising results.
- Work is currently in progress to incorporate finite-β/toroidal/drift effects into our analysis.